Unit I: One-Dimensional Kinematics

A large part of Physics is called mechanics. Mechanics is the study of motion. It is divided into two parts: kinematics and dynamics. Kinematics describes motion, while dynamics looks at the cause of the movement.

Motion

Motion is the movement of an object from one place to another. It is measured over a time interval and is relative to a reference point. It is often best if the reference point is something stationary such as the ground. An object is in motion if its position is changing with respect to an observer.

Frames of Reference

Let's say I am standing on the back of a pickup truck that is motionless and I am throwing apples forward. I know that I can throw an apple at exactly 15 m/s every time. If a person were standing on the sidewalk, how fast would she say the apples are moving? 15 m/s

Now the truck starts moving forwards at 20 m/s. I am still throwing apples forwards, exactly the same as I was throwing them before, at 15 m/s. How fast would the spectator say the apples are moving? 35 m/s

How fast according to me does it look like the apples are moving? 15 m/s

When you are standing on the ground, that is your frame of reference. Anything that you see, watch, or measure will be compared to the reference point of the ground. If I am standing in the back of the moving truck, the truck is now my frame of reference and everything will be measured compared to it. We say that moving objects have relative velocity.

Sitting at your desk, how fast are you moving relative to the ground? 0 m/s Relative to the sun? 30 000 m/s Which answer is correct? Both!

We show different motions as arrows in the direction objects are moving. We call these vectors.

Vectors

While motion can be described with words, that is often not good enough. Physics is a mathematical science, so we use two categories of mathematical quantities to describe motion:

Scalars

-quantities described with a magnitude (or number) only

-Ex. Temperature, mass, and speed

Vectors

-quantities described with both magnitude and direction

-Ex. Velocity, force, acceleration, displacement, weight, momentum

Ex. Which of these is a scalar? A vector?

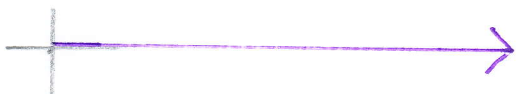
8 m	26.4 m/s East	84°C	52.0 m/s	3 km [N]
S	V	S	S	V

Drawing Vectors to Scale

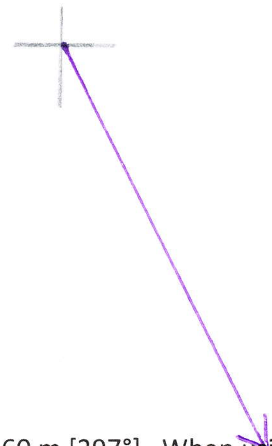
Vectors are used to represent vector quantities on a diagram. A vector is composed of a line segment drawn to scale with an arrowhead at one end. The tail of the vector is at its origin and the head is at the terminal point (the arrowhead). The length of the vector represents its magnitude and the arrowhead indicates its direction. When drawing vectors you must also include reference coordinates. W E

Ex. Draw 24 km [E] to scale

1cm = 4 Km



Ex. Draw 60 m [S27°E]. 1cm = 10m



60 m [S27°E] could also be represent by 60 m [27° E of S] or 60 m [297°]. When using the cardinal directions, your angle should be less than 90°.

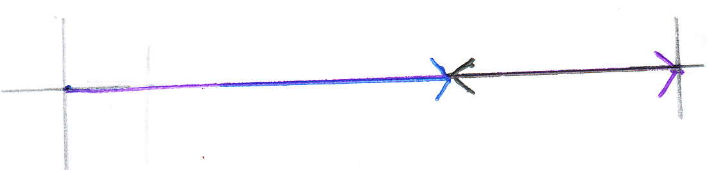
Collinear vectors are vectors that exist in the same dimension. In other words, they exist either in the same direction or in the opposite direction. Non-collinear vectors are vectors that exist in more than one dimension (i.e. they are located along different straight lines).

Adding Vectors Graphically

Add vectors by drawing them head to tail. That is, each vector starts where the last one stopped.

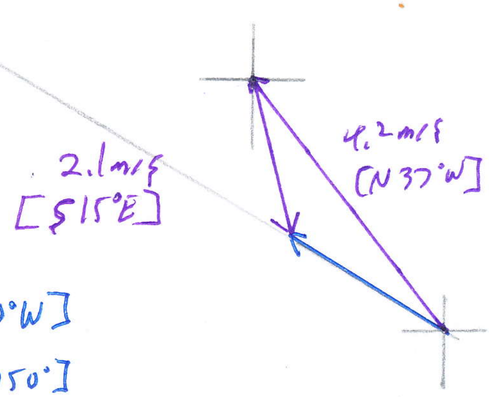
The sum (resultant) is the vector from the tail of the first to the head of the last vector. Measure its length and direction to the same scale.

Ex. 8 m [E] + 3 m [W] 1cm = 1m



$\vec{R} = 5m\,[E]$

Ex. 4.2 m/s [N37°W] + 2.1 m/s [S15°E] 1cm = 1m/s



2.4m/s [N60°W]

or 2.4m/s [150°]

Opposites: $2.7\,m/s\ [N27°E] \rightarrow 2.7\,m/s\ [S27°W]$

$3.6\,m\ [135°] \rightarrow 36\,m\ [135° + 180°] \rightarrow 36\,m\ [315°]$ (If $\theta < 180°$, $+180°$)

$3.6\,m\ [275°] \rightarrow 36\,m\ [275° - 180°] \rightarrow 36\,m\ [95°]$ (If $\theta > 180°$, $-180°$)

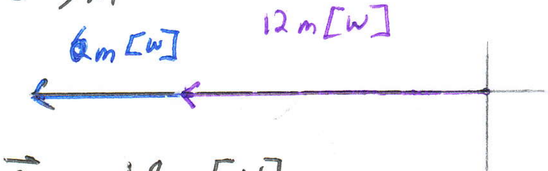
Subtracting Vectors Graphically

Subtracting vectors is like subtracting integers – just add the opposite.

Ex. 12 m [W] – 6 m [E]

12 m [W] + 6 m [W]

1 cm = 3 m

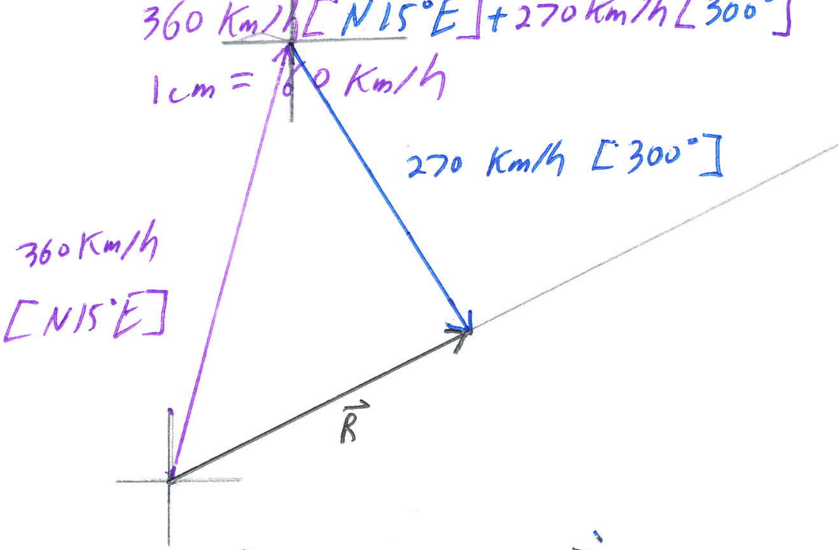


$\vec{R}$ = 18 m [W]

Ex. 360 km/h [15° E of N] – 270 km/h [120°]

360 km/h [N15°E] + 270 km/h [300°]

1 cm = 60 km/h



$\vec{R}$ = 270 km/h [27°]

or $\vec{R}$ = 270 km/h [N63°E]

Mathematical Operations with Vectors

If you multiply a vector by a scalar, you only multiply the magnitude of the vector with the scalar. The direction does not change. The same goes for dividing.

Ex. 2(3.7 m [N23°E])

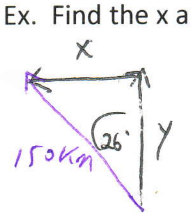
7.4 [N23°E]

Ex. 57 N [32°] ÷ 13

4.4 N [32°]

Vectors can be added mathematically by the vector component method. The vector is broken down into its x and y components. The collinear components are then added together and the Pythagoras theorem is used to find the magnitude. Trigonometry gets you the angle. Remember SOH CAH TOA!

Ex. Find the x and y components of 150 km [N26°W]



$\sin \theta = \frac{x}{h}$

$x = h \sin\theta$

$x = 150\,km(\sin 26°)$

$x = 66\,km\ [W]$

$\cos\theta = \frac{y}{h}$

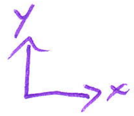
$y = h\cos\theta$

$y = 150\,km(\cos 26°)$

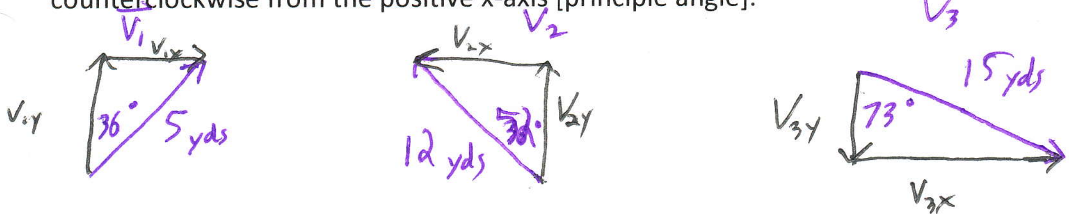
$y = 130\,km\ [N]$

Ex. A football player is hoping to score a touchdown by running a complicated play. He runs 5 yards [N 36° E], then changes his path to run 12 yards [N 52° W] where he meets a block and is forced to run 15 yards [S 73° E] before he is tackled. Determine the resultant vector. Did this player make a successful play?

1 2 3



Step 1: Write down the given information and determine the angle for each vector $\vec{V}$, where θ is measured counterclockwise from the positive x-axis [principle angle].



Step 2: Break each vector down into its x and y components by using a trig ratio. SOH CAH TOA is your friend.

$\vec{V_{1x}} = V_1 \sin\theta$ $\quad \vec{V_{1y}} = V_1 \cos\theta$ $\quad \vec{V_{2x}} = 12yds \sin 52$ $\quad \vec{V_{3x}} = 15yds \sin 73°$

$\vec{V_{1x}} = 5yds \sin 36°$ $\quad \vec{V_{1y}} = 5yds \cos 36°$ $\quad \vec{V_{2x}} = -9.47 yds$ $\quad \vec{V_{3x}} = +14.34 yds$

$\vec{V_{1x}} = 2.94 yds$ $\quad \vec{V_{1y}} = 4.05 yds$ $\quad \vec{V_{2y}} = 12yds \cos 52°$ $\quad \vec{V_{3y}} = 15yds \cos 73°$

$\vec{V_{2y}} = 7.39 yds$ $\quad \vec{V_{3y}} = -4.39 yds$

Step 3: Add the collinear vectors algebraically.

$$\sum \vec{V_x} = \vec{V}_{x1} + \vec{V}_{x2} + \vec{V}_{x3} + \cdots$$

$$\sum \vec{V_y} = \vec{V}_{y1} + \vec{V}_{y2} + \vec{V}_{y3} + \cdots$$

$\vec{V_x} = 2.94yds - 9.47 yds + 14.34 yds$ | $\vec{V_y} = 4.05 yds + 7.39 - 4.39 yds$

$\vec{V_x} = 7.81 yds$ | $\vec{V_y} = 7.05 yds$

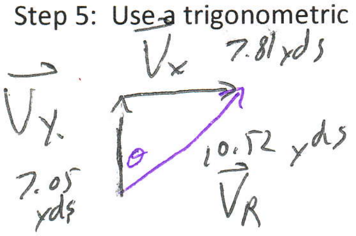
Step 4: Use the Pythagorean theorem to find the magnitude of the resultant vector.

$$\vec{V_R} = \sqrt{\left(\sum \vec{V_x}\right)^2 + \left(\sum \vec{V_y}\right)^2}$$

$\vec{V_R} = \sqrt{(7.81 yds)^2 + (7.05 yds)^2}$ $\qquad \vec{V_R} \approx 10.52 yds$

$\vec{V_R} = \sqrt{110.6986 yds^2}$

Step 5: Use a trigonometric ratio to determine the angle of the resultant vector



$\sin\theta = \dfrac{\vec{V_x}}{\vec{V_R}}$ $\qquad \theta \simeq 48°$

$\sin\theta = \dfrac{7.81 yds}{10.52 yds}$

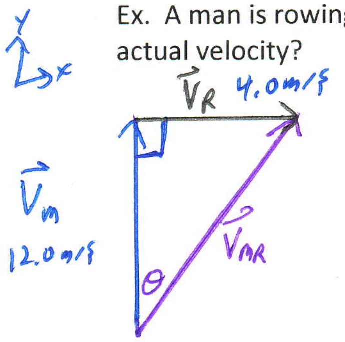
Step 6: Convert this angle into cardinal directions when it makes sense.

[N 48° E] $\qquad \boxed{11 yds [N 48° E]}$ $\qquad$ Successful!

Velocity Vectors

A diagram often goes a long ways in solving physics problems. When dealing with velocity vectors, you need to be sure you are properly representing what is happening. The vector diagram used needs to show which vector is "pushing" which vector.

Ex. A man is rowing across a river. He heads north at 12.0 m/s while the river flows east at 4.0 m/s. What is his actual velocity?



$V_{MR} = \sqrt{V_m^2 + V_R^2}$

$V_{MR} = \sqrt{(12.0\,m/s)^2 + (4.0\,m/s)^2}$

$V_{MR} \approx 13\,m/s$

$\tan\theta = \frac{V_R}{V_m}$

$\tan\theta = \frac{4.0\,m/s}{12.0\,m/s}$

$\theta = 18^\circ$

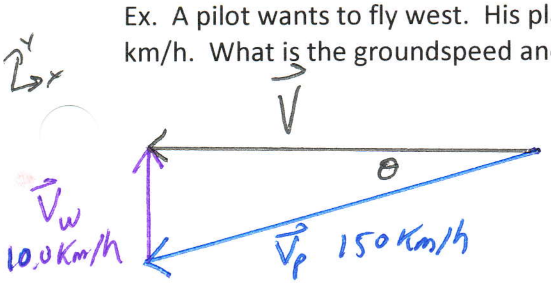
$\vec{V}_m = 12.0\,m/s\,[N]$

$\vec{V}_R = 4.0\,m/s\,[E]$

$\vec{V}_{MR} = ?$

$\therefore \boxed{\vec{V}_{MR} = 13\,m/s\,[N18^\circ E]}$

Ex. A pilot wants to fly west. His plane has an airspeed of 150 km/h. There is a wind blowing north at 10.0 km/h. What is the groundspeed and proper heading?



$a^2 + b^2 = c^2$

$\vec{V}^2 + \vec{V}_w^2 = \vec{V}_p^2$

$\vec{V} = \sqrt{\vec{V}_p^2 - \vec{V}_w^2}$

$\vec{V} = \sqrt{(150\,km/h)^2 - (10.0\,km/h)^2}$

$\vec{V} \approx 150\,km/h$

$\sin\theta = \frac{\vec{V}_w}{\vec{V}_p}$

$\sin\theta = \frac{10.0\,km/h}{150\,km/h}$

$\theta \approx 3.8^\circ$

$\boxed{\vec{V} = 150\,km/h\,[W3.8^\circ S]}$

or 150 km/h [S 86.2°W]

or $\vec{V} = 150\,km/h\,[183.8^\circ]$

Motion

Uniform motion continues in a straight line path at a constant speed. Non-uniform motion is either not in a straight line or not at a constant speed.

Recall that average speed is equal to the distance travelled divided by the time it took.

$v_{av} = \frac{\Delta d}{\Delta t} = \frac{d_2 - d_1}{t_2 - t_1}$	Where v = average speed d1 = initial position d2 = final position Δt = total time taken to travel the distance

Ex. If it took you 2.3 hours to travel the 258.4 km to Saskatoon, at what speed were you travelling?

$t = 2.3\,h$ $v = ?$ $v = \frac{\Delta d}{\Delta t}$ $v = \frac{258.4\,km}{2.3\,h}$

$d = 258.4\,Km$

$\boxed{v \approx 110\,Km/h}$

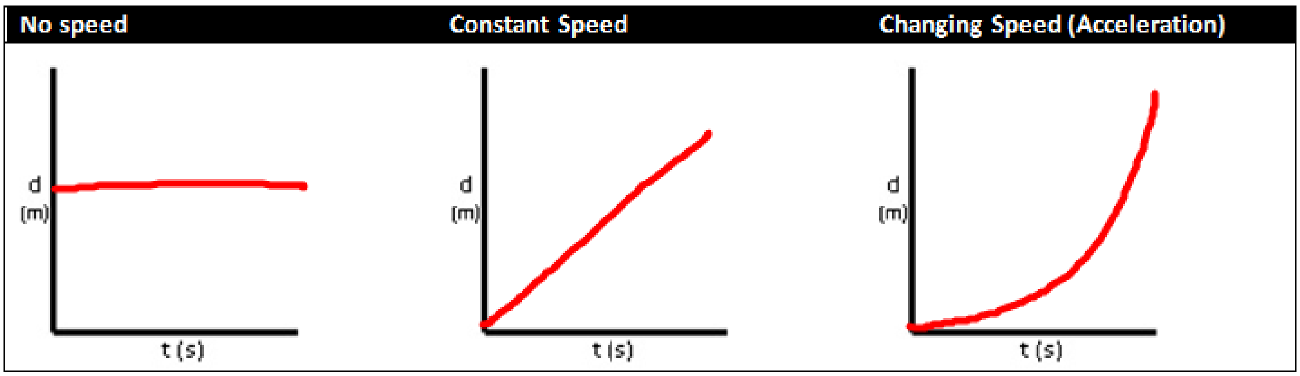
Graphing Motion

Distance-Time Graphs (Position-Time Graphs)

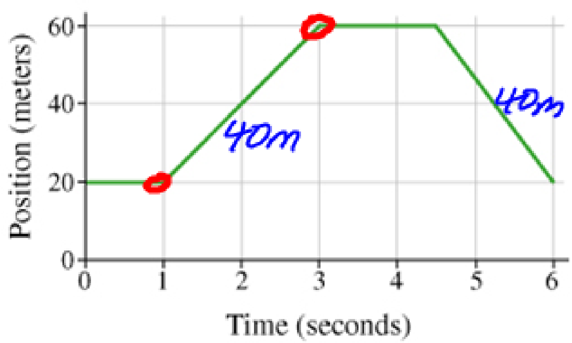
Distance needs no reference frame. You measure the distance between two objects by measuring their separation. It simply refers to the length (or magnitude) between the two objects. Direction does not matter.

The position of an object refers to the change of an objects original starting point to its ending point. In this case, magnitude and direction of motion are important. This means it is a vector unit.

Constant (uniform) motion is given by a straight line on the graph. Curves indicate non-uniform motion. Zero slope represents no motion.



Ex.



A) Find the average speed from 1 to 3 s.

slope: $\frac{\Delta y}{\Delta x} = \frac{60m - 20m}{3s - 1s}$

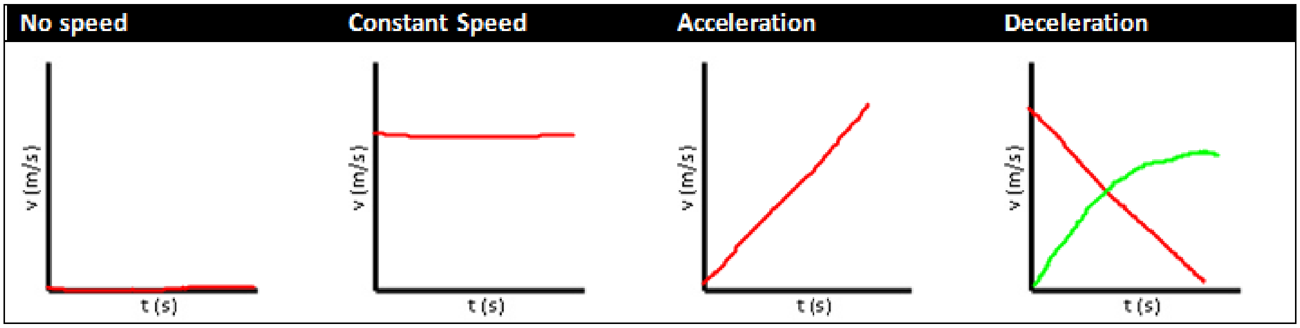
$\approx \boxed{20\,m/s}$

B) What total distance did the object travel?

$40m + 40m = \boxed{80m}$

Speed–Time Graphs

Uniform motion is a zero slope line. A non-zero slope shows constant acceleration.



Deceleration is just a negative acceleration, which would be acceleration in the opposite direction.

Displacement and Velocity Graphs

Displacement refers to an objects change in position from its starting point. It is a vector unit, so the direction is important. To find displacement on a velocity-time graph, find the area under the curve.

Velocity is an object's displacement divided by the period of time the displacement occurred in. It is a vector quantity.

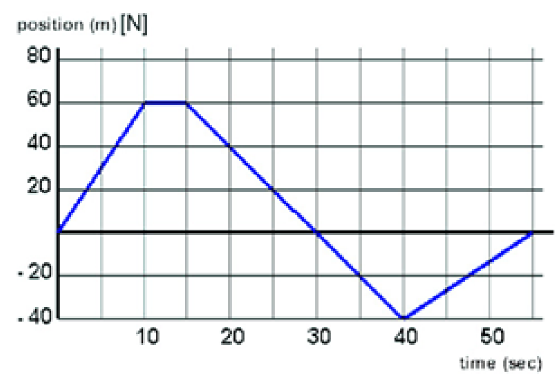
$$\vec{V}_{av} = \frac{\Delta \vec{d}}{\Delta t}$$

Where $\Delta \vec{d} = \vec{d}_2 - \vec{d}_1$ (displacement)

$\Delta t = t_2 - t_1$

$\vec{v}_{av}$ = average velocity

Ex.



a) Find the distance travelled from 10 s to 40 s.

$60m + 40m = 100m$

b) Find the displacement from 10 s to 40 s.

$\vec{\Delta d} = -40m - (+60m)$

$\Delta d = -100m$

or $100m$ [S]

c) Find the average speed from 10 s to 40 s.

$V = \frac{\Delta d}{\Delta t} = \frac{100m}{30s} \approx 3 m/s$

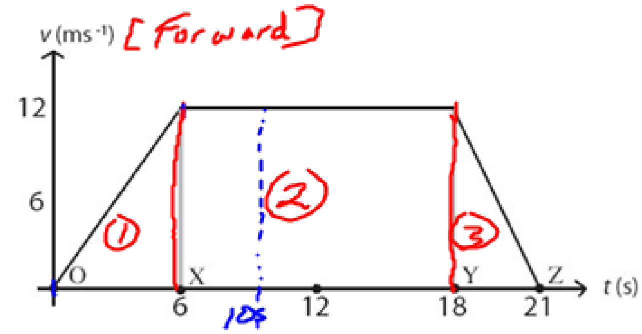
d) Find the average velocity from 10 s to 40 s.

$\vec{V}_{av} = \frac{\vec{\Delta d}}{\Delta t} = \frac{-40m - (60m)}{40s - 10s} = \frac{-100m}{30s} \approx -3 m/s$

$-3 m/s$ [N]

$3 m/s$ [S]

Ex. A moped's velocity, as it is travelling between 2 traffic lights is shown below. Assume a [forward] direction.



a) When was the moped going a uniform velocity?

6–18 s

b) When was the moped accelerating? How can you tell?

0–6 s → non-zero slope
18–21 s → " " "

c) When was the moped decelerating? How can you tell?

18–21 s → negative slope

d) How fast was the moped going at 10 s?

12 m/s

e) Calculate the acceleration of the moped between 18 s and 21 s.

slope = $\vec{a} = \frac{0\,m/s - 12\,m/s}{21\,s - 18\,s} = \frac{-12\,m/s}{3\,s} = \boxed{-4\,m/s^2}$

or $4\,m/s^2$ [backwards]

f) What was the displacement of the moped between 0 s and 21 s? (This is the total distance between the traffic lights)

Area under the curve.

0–6 s: $\frac{12\,m/s(6\,s)}{2} = 36\,m$

6 s – 18 s: $12\,m/s(18-6\,s) = 144\,m$

18 s – 21 s: $\frac{12\,m/s(21-18\,s)}{2} = 18\,m$

$\Delta d = 36\,m + 144\,m + 18\,m$

$\boxed{\Delta d = 198\,m}$

Ex. A leaf blowing in a forward direction. Calculate the displacement from 0 s to 20 s.

① $\frac{0.8(4)}{2} = 1.6\,m$

② $\frac{(9\,s - 4\,s)(-1.0-0)}{2} = -2.5\,m$

③ $\frac{(12-9)(-0.4-(-1))}{2}$ $= -0.9\,m$

④ $(17-9)(-0.4-0)$ $= -3.2\,m$

⑤ $\frac{(-0.4)(1)}{2} = -0.2\,m$

⑥ $\frac{0.8(2)}{2} = 0.8\,m$

$\vec{\Delta d} = 1.6\,m - 2.5\,m - 0.9\,m - 3.2\,m - 0.2\,m + 0.8\,m$

$\boxed{-4.4\,m}$

Acceleration

Acceleration is the rate of change of velocity based on a unit of time. Remember, if acceleration is negative, then the object is decelerating; it is slowing down. If the acceleration is zero, then the object is moving at a constant velocity or may not be moving at all. Since acceleration is a vector, simply changing your direction but not your speed is also a form of acceleration.

$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$	where $\vec{a}_{av}$ = average acceleration $\Delta\vec{v}$ = change in velocity Δt = change in time

final − initial ($\vec{v}_2 - \vec{v}_1$)

Ex. The velocity of a car increases from 2.0 m/s at 1.0 s to 16 m/s at 4.5 s. What is the car's average acceleration?

$V_1 = 2.0\,m/s$
$V_2 = 16\,m/s$
$t_1 = 1.0\,s$
$t_2 = 4.5\,s$
$\vec{a} = ?$

$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$

$\vec{a}_{av} = \frac{V_2 - V_1}{t_2 - t_1}$

$\vec{a}_{av} = \frac{16\,m/s - 2.0\,m/s}{4.5\,s - 1.0\,s}$

$\boxed{\vec{a}_{av} = 4.0\,m/s^2}$

Ex. A car goes faster and faster backwards down a long driveway. The car's velocity changes from -2.0 m/s to -9.0 m/s in a 2.0 s time interval. Find its acceleration.

$V_1 = -2.0\,m/s$
$V_2 = -9.0\,m/s$
$\Delta t = 2.0\,s$
$\vec{a} = ?$

$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$

$\frac{-9.0\,m/s - (-2.0\,m/s)}{2.0\,s}$

$\boxed{\vec{a} = -3.5\,m/s^2}$

Acceleration and Escape Velocity

See handout.

Equations of Motion

Velocity of An Object With Constant Acceleration

Acceleration that does not change in time is uniform or constant acceleration. The slope of the line on a velocity-time graph gives the acceleration. Finding the equation of this line, or just rearranging our average acceleration formula gives: $\vec{a} = \frac{\Delta v}{\Delta t}$

$\boxed{\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t}$	where $\vec{v}_2$ = final velocity $\vec{v}_1$ = initial velocity $\vec{a}$ = acceleration Δt = time interval

Ex. If a car with a velocity of 2.0 m/s at t = 0 accelerates at a rate of 4.0 m/s^2 for 2.5 s what is its velocity at t = 2.5 s?

$\vec{v}_1 = 2.0\ m/s$
$\vec{a} = 4.0\ m/s^2$
$\Delta t = 2.5\ s$
$\vec{v}_2 = ?$

$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$

$\vec{v}_2 = 2.0\ m/s + (4.0\ m/s^2)(2.5\ s)$

$\boxed{\vec{v}_2 \approx 12\ m/s}$

Ex. A car is travelling with a velocity of 1.23 m/s when t = ~~5~~ 5.00 s. It has an unknown constant acceleration, but when t = 10.0 s, it has a velocity of 24.50 m/s. What is the car's acceleration?

$\vec{v}_1 = 1.23\ m/s$
$t_1 = 5.00\ s$
$\vec{v}_2 = 24.50\ m/s$
$t_2 = 10.0\ s$
$\vec{a} = ?$

$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$
$\vec{v}_2 - \vec{v}_1 = \vec{a}\Delta t$
$\frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \vec{a}$

$\vec{a} = \frac{24.50\ m/s - 1.23\ m/s}{10.0\ s - 5.00\ s}$

$\boxed{\vec{a} \approx 4.65\ m/s^2}$

Displacement When Velocity and Time Are Known

To find the displacement if the object is uniformly accelerating, the velocity is replaced by the average velocity.

$$\boxed{\vec{v}_{av} = \frac{\vec{v}_2 + \vec{v}_1}{2}}$$

Using a velocity-time graph, we can find a formula for the area under the curve to give us displacement:

$\vec{v} = \frac{\Delta \vec{d}}{\Delta t} \Rightarrow \Delta \vec{d} = \vec{v}\Delta t$

$$\boxed{\Delta \vec{d} = \frac{1}{2}(\vec{v}_2 + \vec{v}_1)\Delta t}$$

This equation states that displacement is equal to average velocity times time.

Ex. What is the displacement of a train as it is accelerated uniformly from 11 m/s to 33 m/s in a 20.0 s interval?

$\Delta \vec{d} = ?$
$\vec{v}_1 = 11\ m/s$
$\vec{v}_2 = 33\ m/s$
$\Delta t = 20.0\ s$

$\Delta \vec{d} = \frac{1}{2}(\vec{v}_2 + \vec{v}_1)\Delta t$

$\Delta d = \frac{1}{2}(33\ m/s + 11\ m/s)(20.0\ s)$

$\boxed{\Delta \vec{d} \approx 440\ m}$

Ex. Find the initial velocity of the train if the final velocity is 96 km/h and it covered a distance of 730 km in 9.0 h.

$\vec{v}_1 = ?$
$\vec{v}_2 = 96\ km/h$
$\Delta \vec{d} = 730\ km$
$\Delta t = 9.0\ h$

$\Delta d = \frac{1}{2}(\vec{v}_2 + \vec{v}_1)\Delta t$
$2\Delta d = (\vec{v}_2 + \vec{v}_1)\Delta t$
$\frac{2\Delta d}{\Delta t} = \vec{v}_2 + \vec{v}_1$
$\frac{2\Delta d}{\Delta t} - \vec{v}_2 = \vec{v}_1$

$\vec{v}_1 = \frac{2(730\ km)}{9.0\ h} - 96\ km/h$

$\boxed{\vec{v}_1 \approx 66\ km/h}$

Displacement When Acceleration and Time are Known

By combining two of the above equations, we get the following:

$$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$

If acceleration is zero, this equation simply turns into d = vt.

We can also get:

$$\Delta\vec{d} = \vec{v}_2 t - \frac{1}{2}\vec{a}(\Delta t)^2$$

Ex. A car starting from rest accelerates uniformly at 6.1 m/s^2 for 7.0 s. How far does the car move?

$\vec{v}_1 = 0\,m/s$
$\vec{a} = 6.1\,m/s^2$
$\Delta t = 7.0\,s$
$\Delta\vec{d} = ?$

$\Delta\vec{d} = \cancel{\vec{v}_1\Delta t} + \frac{1}{2}\vec{a}(\Delta t)^2$
$\Delta\vec{d} = \frac{1}{2}(6.1\,m/s^2)(7.0\,s)^2$
$\Delta\vec{d} = 149.45\,m$
$\boxed{\Delta\vec{d} \approx 150\,m}$

Ex. Find the acceleration of an object when the initial velocity is 12 m/s, the time is 7.0 s, and the displacement is 382 m.

Displacement When Velocity and Acceleration Are Known

Again, combining equations from above gives us another equation:

$$\vec{v}_2^2 = \vec{v}_1^2 + 2\vec{a}\Delta\vec{d}$$

This is useful when you don't know the time interval the motion occurred over.

Ex. Find the final velocity of an object if the initial velocity is 17 m/s, the acceleration is 3.4 m/s^2, and the displacement is 1400 m.

$\vec{v}_2 = ?$
$\vec{v}_1 = 17\,m/s$
$\vec{a} = 3.4\,m/s^2$
$\Delta\vec{d} = 1400\,m$

$\vec{v}_2^2 = \vec{v}_1^2 + 2\vec{a}\Delta\vec{d}$
$\vec{v}_2 = \sqrt{\vec{v}_1^2 + 2\vec{a}\Delta\vec{d}}$
$\vec{v}_2 = \sqrt{(17\,m/s)^2 + 2(3.4\,m/s^2)(1400\,m)}$
$\boxed{\vec{v}_2 \approx 99\,m/s}$

Ex. An airplane must reach a velocity of 71 m/s for takeoff. If the runway is 1.0 km long, what must the constant acceleration be?

$\vec{v}_2 = 71\,m/s$

$\Delta d = 1.0\,km = 1000\,m$

$\vec{a} = ?$

$\vec{v}_1 = 0\,m/s$

$\vec{v}_2^{\,2} = \cancel{\vec{v}_1^{\,2}}^{\,0} + 2\vec{a}\Delta\vec{d}$

$\dfrac{\vec{v}_2^{\,2}}{2\Delta\vec{d}} = \vec{a}$

$\vec{a} = \dfrac{(71\,m/s)^2}{2(1000\,m)}$

$\boxed{\vec{a} \simeq 2.5\,m/s^2}$

Ex. Jack and Jill ran down the hill. Both started from rest and accelerated steadily. Jack accelerated at 0.25m/s^2 and Jill at 0.30 m/s^2. After running 20.0 s, Jill fell down.

a) How far did Jill get before she fell?

$\Delta\vec{d} = ?$

$\vec{v}_1 = 0\,m/s$

$\vec{a} = 0.30\,m/s^2$

$\Delta t = 20.0\,s$

$\Delta\vec{d} = \cancel{\vec{v}_1\Delta t} + \frac{1}{2}\vec{a}(\Delta t)^2$

$\Delta\vec{d} = \frac{1}{2}\vec{a}(\Delta t)^2$

$\Delta\vec{d} = \frac{1}{2}(0.30\,m/s^2)(20.0\,s)^2$

$\Delta\vec{d} \simeq 6\bar{0}\,m$

b) How far had Jack travelled when Jill fell?

$\Delta\vec{d} = \frac{1}{2}\vec{a}(\Delta t)^2$

$\Delta\vec{d} = \frac{1}{2}(0.25\,m/s^2)(20.0\,s)^2$

$\boxed{\Delta\vec{d} = 5\bar{0}\,m}$

c) How fast was Jack running when Jill fell?

$\vec{v}_2 = ?$

$\vec{v}_1 = 0\,m/s$

$\vec{a} = 0.25\,m/s^2$

$\Delta\vec{d} = 50\,m$

$\Delta t = 20.0\,s$

$\vec{v}_2^{\,2} = \cancel{\vec{v}_1^{\,2}}^{\,0} + 2\vec{a}\Delta d$

$\vec{v}_2 = \sqrt{2\vec{a}\Delta d}$

$\vec{v}_2 = \sqrt{2(0.25\,m/s^2)(50\,m)}$

$\boxed{\vec{v}_2 \simeq 5.0\,m/s}$

d) How long was it after Jill fell that Jack ran into her and broke his crown (to the nearest second)?

$\Delta\vec{d} = 60\,m - 50\,m = 10\,m$

$\vec{v}_1 = 5.0\,m/s$

$\vec{a} = 0.25\,m/s^2$

$\Delta t = ?$

$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$

$10\,m = 5.0\,m/s\,\Delta t + \frac{1}{2}(0.25\,m/s^2)(\Delta t)^2$

$0 = \underbrace{-10\,m}_{c} + \underbrace{5.0\,m/s}_{b}\Delta t + \underbrace{\tfrac{1}{2}(0.25\,m/s^2)}_{a}(\Delta t)^2$

Quadratic:

$\Delta t = \dfrac{-5\,m/s \pm \sqrt{(5\,m/s)^2 - 4(0.125)(-10)}}{2(0.125)}$

$\boxed{\Delta t = 2\,s}$, $\cancel{-42\,s}$